Inside and outside bark taper equations for white spruce and white pine plantations

Technical Report TR-58

Science and Research Branch
Ministry of Natural Resources and Forestry

Ontario

Science and Research Technical Report TR-58

Inside and outside bark taper equations for white spruce and white pine plantations

Mahadev Sharma

Forest Research and Monitoring Section, 1235 Queen St. East, Sault Ste Marie, ON

2023

Science and Research Branch

Ministry of Natural Resources and Forestry

Copies of this publication are available from info.mnrfscience@ontario.ca.

Cette publication hautement spécialisée, Inside and Outside Bark Taper Equations for White Spruce and White Pine Plantations, n'est disponible qu'en anglais conformément au Règlement 671/92, selon lequel il n'est pas obligatoire de la traduire en vertu de la Loi sur les services en français. Pour obtenir des renseignements en français, veuillez communiquer avec le ministère des Richesses naturelles et des Forêts au info.mnrfscience@ontario.ca.

Some of the information in this document may not be compatible with assistive technologies. If you need any of the information in an alternate format, contact info.mnrfscience@ontario.ca.

Cite this report as:

Sharma, M. 2023. Inside and outside bark taper equations for white spruce and white pine plantations. Ontario Ministry of Natural Resources and Forestry, Science and Research Branch, Peterborough, ON. Science and Research Technical Report TR- 58. 20 p.

Abstract

Taper equations are used to estimate tree diameter at any height along the bole. Individual tree volumes and product recoveries can be calculated based on these diameters and corresponding heights. Tree diameters are affected by stand establishment method — natural stand vs. plantation — and the resulting stand structure and density. The goal of this study was to develop inside and outside bark taper equations for white spruce and white pine plantations by incorporating stand density information. Data was obtained by sampling 200 trees from 40 even-aged monospecific plantations per species across Ontario, Canada.

A dimensionally compatible taper equation was adapted and fitted using a nonlinear mixed-effects model. Quadratic mean diameter (QMD) explained most of the variation in stem taper of plantation grown white spruce and white pine trees. Crown ratio was significant without stand density for white spruce but was nonsignificant in the presence of QMD. For white pine, however, crown ratio was nonsignificant with or without QMD. The taper models presented here are dimensionally compatible and can therefore be applied to data in any unit system without adjusting estimated parameter values.

Résumé

Équations de défilement de l’écorce interne et externe pour les plantations d’épinettes blanches et de pins blancs

Les équations de défilement servent à estimer le diamètre d’un arbre à n’importe quelle hauteur du tronc. Les volumes des arbres individuels et les rétablissements du produit peuvent être calculés en fonction de ces diamètres et des hauteurs correspondantes. Les diamètres des arbres sont affectés par la méthode d’établissement du peuplement — un peuplement naturel par rapport à une plantation, la structure du peuplement qui en résulte et sa densité. L’objectif de cette étude consiste à élaborer des équations de défilement de l’écorce interne et externe pour les plantations d’épinettes blanches et de pins blancs en intégrant l’information sur la densité du peuplement. Les données ont été obtenues en échantillonnant 200 arbres provenant de 40 plantations équiennes monospécifiques par espèce à l’échelle de l’Ontario, Canada.

Une équation de défilement dimensionnellement compatible a été adoptée et adaptée à l’aide d’un modèle non linéaire à effets mixtes. Le diamètre de la tige de surface terrière moyenne expliquait en grande partie la variation du défilement du tronc des épinettes blanches et des pins blancs cultivés. Le rapport/cime hauteur totale était important sans la densité du peuplement pour l’épinette blanche, mais était non significatif en présence du diamètre de la tige de surface terrière moyenne. Dans le cas du pin blanc, le rapport/cime hauteur totale était cependant non significatif avec ou sans le diamètre de la tige de surface terrière moyenne. Les modèles de défilement présentés ici sont dimensionnellement compatibles et peuvent par conséquent être appliqués aux données dans tout système d’unités sans rajustement des valeurs des paramètres estimées.

Acknowledgements

This study was supported by the Forest Futures Trust Knowledge Transfer and Tool Development (KTTD) program and the Ontario Ministry of Natural Resources and Forestry (MNRF). The author is grateful to Chris Stratton and Todd Little, Biodiversity and Monitoring Section, MNRF, for coordinating data collection work and producing sample location map, respectively.

Contents

Introduction

Stem profile equations or taper equations are used to estimate tree diameter at any given height along the bole. Given corresponding heights, these diameters can be used to calculate total volume and product recoveries from an individual tree. Depending on their intended use, Volumes are calculated as outside or inside bark. Outside bark volume estimates are used to calculate tree biomass and carbon stocks, while inside bark volume estimates are used for merchantable volume and product recovery.

Volume and product recovery depend on tree shape. Trees from natural stands are more parabolic than those grown in plantations (Sharma and Parton 2009) and therefore contain more volume and provide higher product recovery than trees grown in plantations for a given diameter at breast height (DBH) and total height (Sharma and Burkhart 2003). Thus, plantation grown trees offer less economic value than those grown in natural stands (Sharma 2020).

Stand density affects tree form regardless of stand type (Gray 1956, Larson 1963, Sharma and Parton 2009, Sharma 2020). Given similar environmental and stand conditions, taper profiles vary among tree species. This variation has been reported for black spruce (*Picea mariana* (Mill.) B.S.P.), jack pine (*Pinus banksiana* Lamb.), and balsam fir (*Abies balsamea* (L.) Mill.) trees grown in natural stands (Sharma and Zhang 2004). Similarly, taper profiles of jack pine, black spruce, and red pine (*Pinus resinosa*) are known to vary among plantations depending on stand densities (Sharma and Parton 2009, Sharma 2020).

Stand density can be regulated by adjusting tree spacing during planting in plantations or by thinning to different densities in natural stands. However, plantations grown to a particular density and natural stands thinned to the same density may not have the same taper profiles, especially if thinning occurs at a later age (Sharma and Zhang 2004). Therefore, taper profiles from different environmental and stand conditions cannot be accurately described using a simple mathematical model, but rather require multiple models of varying complexity.

Researchers have used three approaches to advance these models. The first relies on a simple mathematical function to describe the taper of the whole tree (e.g., Kozak et al. 1969, Ormerod 1973, Amidon 1984, Reed and Byrne 1985, Sharma and Oderwald 2001, Sharma et al. 2002). The second involves dividing the tree stem into three segments approximated by three geometric solids: neiloid (the lower bole), paraboloid (the middle), and cone (the upper bole) (Avery and Burkhart 2001). Each segment is then described by a mathematical function and these functions are combined to create a single model called a segmented polynomial taper model (see Max and Burkhart 1976, Demaerschalk and Kozak 1977, Cao et al. 1980, Fang et al. 2000, Sharma and Burkhart 2003). The third involves assuming tree form varies along the bole and a single continuous mathematical function can be used to describe this variation (Kozak 1988, Newnham 1992, Flewelling and Raynes 1993, Sharma and Zhang 2004). All approaches use DBH and total height as covariates, while some also include crown dimensions (e.g., crown diameter, height, ratio), site class, breast-height age, and quadratic mean diameter (QMD), although adding crown dimensions did not significantly improve model performance (Burkhart and Walton 1985, Valenti and Cao 1986, Newnham 1992, Muhairwe et al. 1994, Leites and Robinson 2004).

Nevertheless, site and stand conditions affect individual tree crown dynamics and taper (Sharma 2020). In fact, stand density information (trees and basal area (BA) per unit area) is obtained without additional cost during a forest inventory. These stand density terms have therefore been used as covariates in taper profile modelling for jack pine, black spruce, and balsam fir, and improved fit statistics and predictive accuracy for these species (Sharma and Zhang 2004, Sharma and Parton 2009). The inclusion of these metrics and stand planting density has therefore become increasingly common in taper profile modelling globally (Duan et al. 2016, Sanquetta et al. 2020, Sharma 2020).

White spruce (*Picea glauca*) is primarily a boreal tree species occurring throughout much of Canada and is commonly planted across Ontario. White pine (*Pinus strobus* L.) is found in most of Ontario, except the Far North, but is planted primarily in the central and southern parts of the province. The objectives of this study were to (1) use mixed-effects models to develop inside and outside bark taper models for white spruce and white pine by incorporating stand density and/or crown ratio and (2) examine the effect of stand density and crown ratio on taper of white spruce and white pine trees grown in plantations.

Methods

Data

Data for this study was obtained by sampling 80 even-aged pure white spruce and white pine plantations (40 plantations/species) across Ontario, Canada, with a variable size circular temporary plot (TSP) established in each plantation (Figure 1). The minimum plot size was 400 m^2, but when necessary was increased to include a minimum of 40 trees. Only planted trees with no visible deformities (e.g., major stem injuries, forked, or dead, or broken tops) were sampled. Five sample trees were collected from each plot for a total of 200 trees per species across all sample locations.

Sampled trees were measured using Ontario's growth and yield assessment standards (Hayden et al. 1995). Total basal area (BA ha^{-1}) and stem density (trees ha^{-1}) were calculated for all live trees in each plot. Cumulative basal area was determined by sequentially numbering all trees of target species growing in the plot. Total cumulative basal area of each plot was divided into five classes based on diameter distributions. One tree from each class was randomly selected for destructive sampling. Mean, standard deviation, and minimum and maximum values for all sampled trees were calculated for DBH, tree height, crown ratio, BA, stem density, and quadratic mean diameter (QMD) for both species (Table 1).

Each sampled tree was cut at three heights below breast height (0.15, 0.5, 0.9 m) and at breast height (1.3 m) to assess diameter growth rate. The tree was also cut at nine equally spaced heights above breast height, resulting in 13 cuts per tree.

Figure 1. Distribution of white spruce (Sw) and white pine (Pw) plantation sites sampled across Ontario, Canada.

The largest outside and inside bark diameters and the ones perpendicular to them and passing through the pith were measured at each stem height where sections were cut. Mean inside and outside bark diameters were obtained by averaging these diameters at that stem height for each tree per species.

Taper models

Sharma and Parton (2009) derived a variable exponent taper model for jack pine and black spruce plantations based on the dimensionally compatible taper model originally presented by Sharma and Oderwald (2001). The mathematical form of this model was:

$$\frac{d}{D} = \beta_0 \left(\frac{H-h}{H-h_D}\right)\left(\frac{h}{h_D}\right)^{\beta_1+\beta_2 x+\beta_3 x^2} + \varepsilon \qquad (1)$$

where d is the diameter inside or outside bark at any given height, h (m), D is the diameter at breast height (DBH) of the outside bark (cm), H is the total tree height (m) from ground to tip, h_D is the breast height (m), x is tree height divided by diameter at breast height of the outside bark *(h/H)*, ε is the error term, and β_i (i=0, 1, 2, and 3) are parameters.

Table 1. Summary statistics for sampled white spruce and white pine plantation trees from across Ontario used in this study. DBH=diameter at breast height; height=total height; BA=basal area; trees ha^{-1}=stem density; QMD=quadratic mean diameter; Std Dev=standard deviation.

Species/attribute	N	Mean	Std Dev	Minimum	Maximum
White spruce					
DBH (cm)	200	24.83	7.03	10.10	48.80
Height (m)	200	19.59	3.08	12.30	26.75
Crown ratio	200	0.453	0.127	0.181	0.804
BA (m^2 ha^{-1})	40	41.65	11.16	22.76	81.50
Trees ha^{-1}	40	1134	451	533	2625
QMD (cm)	40	22.46	4.70	14.90	38.97
White pine					
DBH (cm)	200	27.78	8.84	11.50	55.10
Height (m)	200	21.09	4.59	8.60	34.90
Crown ratio	200	0.409	0.132	0.139	0.913
BA (m^2 ha^{-1})	40	44.00	12.15	23.09	78.84
Trees ha^{-1}	40	975	452	367	2425
QMD (cm)	40	25.36	5.83	15.40	38.31

Sharma and Parton (2009) found that stand density significantly affects taper of jack pine and black spruce in plantations and therefore included stand density information in their model. The model with stand density is written as:

$$\frac{d}{D} = \beta_0 \left(\frac{H-h}{H-h_D}\right) \left(\frac{h}{h_D}\right)^{\beta_1+\beta_2 x+\beta_3 x^2+\beta_4 f(sd)} + \varepsilon \qquad (2)$$

where f(sd) is a function of stand density, β_4 is the parameter to be estimated, and other variables are as defined above. To keep the dimensionless property of the model, the function f(sd) in equation (2) should be dimensionless. Therefore, they evaluated several dimensionless functions of stand density and found that f(sd) =$\sqrt{BA}$/D best explained the variation in taper for jack pine and black spruce in plantations. This model was also used for red pine plantations and the combined stand density, defined as the square root of basal area per hectare divided by trees per hectare $\left(\sqrt{\frac{BA}{TPH}}\right)$, explained the most variation in taper (Sharma 2020).

Equation (2) can be further modified to include crown ratio (live crown length/total tree height). The taper model with stand density and crown ratio can be expressed as:

$$\frac{d}{D} = \beta_0 \left(\frac{H-h}{H-h_D}\right) \left(\frac{h}{h_D}\right)^{\beta_1 + \beta_2 x + \beta_3 x^2 + \beta_4 f(sd) + \beta_5\, CR} + \varepsilon \qquad (3)$$

where, CR is the crown ratio and β_5 is the parameter to be estimated.

Model selection and fitting

Equations (1)–(3) were evaluated for white spruce and white pine trees. These equations, without random effects, were fit to inside bark taper data for each species using NLMIXED procedure in SAS (SAS Institute Inc. 2004). The fit statistic, mean square error (σ^2; MSE), and Akaike information criterion (AIC) values were assessed to determine models of best fit for each species. NLMIXED procedure was selected to account for autocorrelation among observations between trees (Pinheiro and Bates 1995, Demidenko 2004, Sharma and Parton 2009, Sharma 2020).

The effects of stand density were modelled by incorporating dimensionless functions of stand density, including (1) QMD/D, (2) BA/D2, (3)$\sqrt{BA}$/D, and (4) tree density (number of trees/hectare (TPH)), one at a time into the models. Since QMD is obtained by multiplying $\sqrt{\frac{BA}{TPH}}$ and a constant (112.8379), in this study QMD was used instead of $\sqrt{\frac{BA}{TPH}}$. The function of best fit was incorporated into the model to account for stand density effects on taper of white spruce and white pine trees.

Stand density and crown ratio variables were introduced in the model one at a time. Random effects parameters were added sequentially to the fixed-effects coefficients as necessary. To assess heteroscedasticity, estimates of residuals (observed − predicted) from the taper model were calculated at each disk cut location and plotted against predicted inside and outside bark diameters.

The effect of stand density on taper for white spruce and white pine trees was evaluated by estimating inside bark and outside bark diameters along the bole of all trees using the model with a stand density term (Eq. 2). To determine the effect of crown ratio on model predictive accuracy, diameters were calculated using the model fitted with crown ratio (Eq. 3). Crown ratio was only included if the term was significant in the presence of stand density. Model accuracy for estimating diameters was evaluated by analyzing residuals for both inside and outside bark diameters. Residuals were calculated by subtracting estimated from observed diameters for all models for both species.

Predictive accuracy of the models for the entire length of the stem was determined by dividing the relative height of each tree from each species into 10 sections. Bias and standard deviation were calculated and compared at each section for both the inside and outside bark models. Finally, the effects of stand density and crown ratio were visually inspected by producing tree profiles (mean responses) using the models with stand density and crown ratio terms for both inside and outside bark diameters for both tree species.

Results

The stand density terms, QMD/D, BA/D^2, $\sqrt{BA}$/D, and stem density, were each significant when incorporated as the stand density term in equation (2) and fitted with white spruce and white pine inside and outside bark taper data. The model with QMD/D had the lowest AIC value for both inside and outside bark diameters for both species. Models fit with BA/D^2, $\sqrt{BA}$/D, and stem density in the presence of QMD/D were nonsignificant. Therefore, QMD/D was included as the stand density term in the taper model (Eq. 2) for both tree species.

Crown ratio was added in the presence of stand density (QMD/D) in equation (3) and fitted with inside and outside bark diameters for both species. For white spruce, both stand density and crown ratio were significant in the regression (Table 2). For white pine, the crown ratio term was nonsignificant in the presence and absence of QMD/D (Table 3). AIC decreased when stand density term, QMD/D, was included in equation 3 and decreased further for white spruce only when crown ratio was also included.

Table 2. Parameter estimates and fit statistics (σ^2=MSE and AIC=Akaike's information criterion) for equations (1)–(3) fitted to inside bark diameters of white spruce trees from across Ontario. NA=not available.

Parameters	Equation 1		Equation 2		Equation 3	
	Estimates	SE	Estimates	SE	Estimates	SE
β_0	0.98100	0.00179	0.98080	0.00175	0.98090	0.00173
β_1	-0.05592	0.00138	-0.09067	0.00327	-0.06096	0.00552
β_2	0.43620	0.01222	0.43830	0.01194	0.43660	0.01185
β_3	-0.16570	0.01530	-0.16700	0.01496	-0.16510	0.01483
β_4	NA	NA	0.04143	0.00358	0.02562	0.00356
β_5	NA	NA	NA	NA	-0.04189	0.00628
σ^2	0.00218	0.00006	0.00208	0.00006	0.00205	0.00006
AIC	-9154	NA	-9283	NA	-9325	NA

Random effects were added to the fixed-effects parameters starting at β_0 at tree level. AIC for models without stand density and crown ratio (Eq. 1) decreased with the addition of random effects for parameters β_0, β_1, β_2, and β_3 for both inside and outside bark diameters. The random effect for parameter β_4 was not significant. The random effect for parameter β_3 was nonsignificant for inside and outside bark diameters for white spruce when QMD/D was included in the model. The four random effects were significant for inside and outside bark diameters of white pine when QMD/D was included.

Table 3. Parameter estimates and fit statistics (σ^2=MSE and AIC=Akaike's information criterion) for equations (1) and (2) fitted to inside bark diameters of white pine trees from across Ontario. NA= not available.

Parameters	Equation 1		Equation 2	
	Estimates	SE	Estimates	SE
β_0	0.93750	0.00158	0.93740	0.00156
β_1	-0.03224	0.00130	-0.05013	0.00267
β_2	0.46590	0.01101	0.46730	0.01090
β_3	-0.2506	0.01372	-0.25160	0.01358
β_4	NA	NA	0.01833	0.00241
σ^2	0.00174	0.00005	0.001709	0.00005
AIC	-9835	NA	-9890	NA

Crown ratio was nonsignificant in the model when fit with stand density and random effect parameters β_0, β_1, and β_2 for inside and outside bark diameters for white spruce. AIC values were not significantly different for white spruce models with stand density and crown ratio fit separately for both inside and outside bark diameters.

Random effects were added to β_1, β_2, β_3, and β_4 at the diameter position along the bole of trees as these parameters determine tree shape. Regressions using these random effects at this level did not converge for either species. Therefore, random effects were used only at tree level.

The final models for inside and outside bark diameter with stand density and random effects are written as:

White spruce

$$y_{ijk} = (\beta_0 + b_{0ij}) \left(\frac{H_{ij}-h_{ijk}}{H_{ij}-h_D}\right) \left(\frac{h_{ijk}}{h_D}\right)^{(\beta_1+b_{1ij})+(\beta_2+b_{2ij})x_{ijk}+\beta_3 x_{ijk}{}^2+\beta_4\frac{QMD_i}{D_{ij}}} + \varepsilon_{ijk} \quad (4)$$

White pine

$$y_{ijk} = (\beta_0 + b_{0ij}) \left(\frac{H_{ij}-h_{ijk}}{H_{ij}-h_D}\right) \left(\frac{h_{ijk}}{h_D}\right)^{(\beta_1+b_{1ij})+(\beta_2+b_{2ij})x_{ijk}+(\beta_3+b_{3ij})x_{ijk}{}^2+\beta_4\frac{QMD_i}{D_{ij}}} + \varepsilon_{ijk} \quad (5)$$

Where, y_{ijk}=d/D at the k^{th} ($k=1, . . ., 13$) observation (diameter measurement) along the bole of the j^{th} ($j=1, . . ., 5$) tree at plot i ($i=1, 2, 3, \ldots 40$). H_{ij}=total height of j^{th} tree at i^{th} plot, h_{ijk}=height at kth observation of j^{th} tree at i^{th} plot, b_{lij} is the l^{th} random effect parameter (l=0, 1, 2, 3) for plot i and tree j, and h_D is breast height (1.3 m unless DBH is measured at another height).

The final models were fit using NLMIXED procedure with three and four random effects for white spruce (Eq. (4)) and white pine (Eq. (5)), respectively, for both inside and outside bark diameters (tables 4 and 5). All parameters including random effects were significant ($p<0.05$) except the covariances between β_0 and β_1, β_2, β_3 for both inside and outside bark diameters for white pine (tables 4 and 5).

Table 4. Parameter estimates and fit statistics for equation (4) estimated using NLMIXED procedure in SAS for inside and outside bark diameters of white spruce trees grown in plantations across Ontario. NA=not available.

	Inside bark		**Outside bark**	
Parameters	**Estimates**	**SE**	**Estimates**	**SE**
β_0	0.98050	0.00147	1.00680	0.00127
β_1	-0.07500	0.00799	-0.07455	0.00777
β_2	0.43900	0.00772	0.43810	0.00763
β_3	-0.17080	0.00778	-0.15940	0.00773
β_4	0.02070	0.00816	0.01861	0.00793
Variance components				
σ^2	0.00055	0.00002	0.00058	0.00002
var (b_0)	0.00026	0.00004	0.00015	0.00003
var (b_1)	0.00070	0.00007	0.00066	0.00007
var (b_2)	0.00415	0.00048	0.00394	0.00046
cov(b_0, b_1)	-0.00017	0.00004	-0.00013	0.00003
cov(b_0, b_2)	0.00019	0.00009	0.00009	0.00008
cov(b_1, b_2)	-0.00077	0.00015	-0.00074	0.00014
AIC	-11591	NA	-11544	NA

The estimate for parameter β_0 is less than one for inside bark diameter for both species, greater than one for white spruce outside bark diameter, and less than one for white pine outside bark diameter (Table 5). The 95% confidence limits of the estimate for outside bark did not include the theoretical value of one for either species. This outcome results from outside bark DBH being measured with a tape rather than calipers, giving larger measurements for white spruce and smaller ones for white pine. β_1 and β_3 estimates were negative and β_2 and β_4 estimates were positive for both inside and outside bark diameters for both species (tables 4 and 5).

As mentioned earlier, the stand density term (QMD/D) and crown ratio were very competitive in both inside and outside bark taper models in terms of fit statistics (AIC and MSE) for white spruce. To compare predictive accuracies of taper models with the stand density term and crown ratio, equation (4) was fitted by replacing the stand density term with crown ratio for this tree species. Residual plots were made by calculating bias in estimating diameters along the boles of white spruce trees using equation (4) with the stand density term and crown ratio separately in the models.

Trends in error structure did not suggest any further autocorrelation or heteroscedasticity for white spruce inside bark (Figure 2) or outside bark diameters (Figure 3). For trees with larger diameters, the error distribution was tighter for the model with the stand density term than that with crown ratio for both inside and outside bark diameters (figures 2 and 3). None of the trends in error structure for white pine inside and outside bark diameters suggest any further autocorrelation or heteroscedasticity (Figure 4). Models with crown ratio are not displayed because the term was non-significant for this species in equation (5).

Table 5. Parameter estimates and fit statistics for equation (5) estimated using NLMIXED procedure in SAS for inside and outside bark diameters of white pine trees grown in plantations across Ontario. NA=not available.

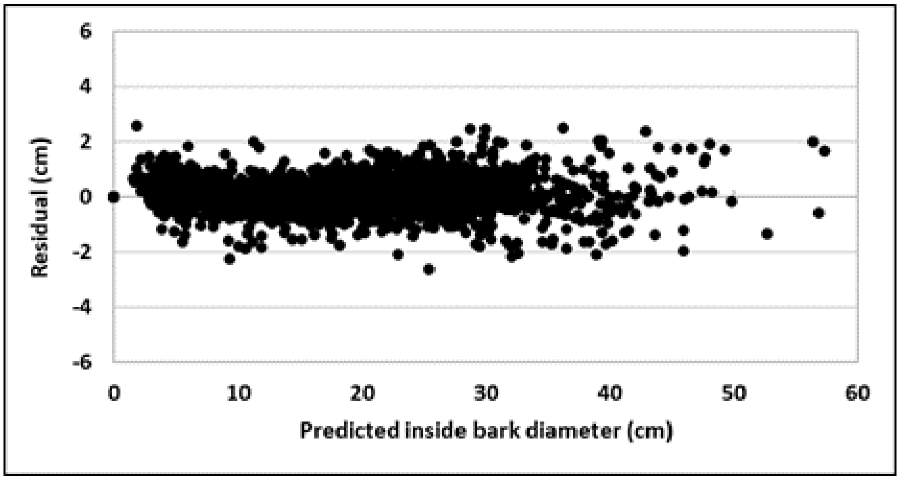
	Inside bark		Outside bark	
Parameters	**Estimates**	**SE**	**Estimates**	**SE**
β_0	0.93620	0.00143	0.99350	0.00116
β_1	-0.05525	0.00514	-0.06736	0.00512
β_2	0.48900	0.01292	0.49800	0.01210
β_3	-0.28370	0.01770	-0.26660	0.01667
β_4	0.02279	0.00509	0.02046	0.00509
Variance components				
σ^2	0.00045	0.00001	0.00051	0.00002
var (b_0)	0.00046	0.00006	0.00033	0.00003
var (b_1)	0.00031	0.00004	0.00031	0.00004
var (b_2)	0.02814	0.00356	0.02502	0.00298
var (b_3)	0.05423	0.00671	0.04744	0.00574
cov(b_0, b_1)	0.00001*	0.00004	0.00000*	0.00003
cov(b_0, b_2)	-0.00030*	0.00033	-0.00042*	0.00029
cov(b_0, b_3)	0.00049*	0.00045	0.00058*	0.00039
cov(b_1, b_2)	-0.00097	0.00032	-0.00088	0.00029
cov(b_1, b_3)	0.00091	0.00043	0.00080	0.00039
cov(b_2, b_3)	-0.03651	0.00474	-0.03195	0.00425
AIC	-11921	NA	-11854	NA

Note: *=not significant at α=0.05

Biases in estimating white spruce diameters are small and similar for models with stand density and crown ratio across relative height for both inside and outside bark diameters for white spruce (Table 6). The biases in estimating white pine diameters are smaller for the model with than for the one without a stand density term (Table 7).

The effects of stand density on tree taper were visually inspected by creating tree profiles (mean responses) using equations (4) and (5) (without random parameters) for white spruce and white pine, respectively. Average DBH of 25.0 cm, total height of 20 m, and QMD of 10 cm, 25 cm, and 40 cm were applied to equation (4). Average DBH of 28.0 cm, total height of 21 m, and QMD of 10 cm, 25 cm, and 40 cm were applied to equation (5). The effect of stand density was similar for inside and outside bark diameters for both species. Stem diameters above breast height are larger at higher QMD and below breast height are smaller at higher QMD (figures 5 and 6). For both species, stand density effect on taper is therefore positive above breast height and negative below breast height.

(a)



(b)

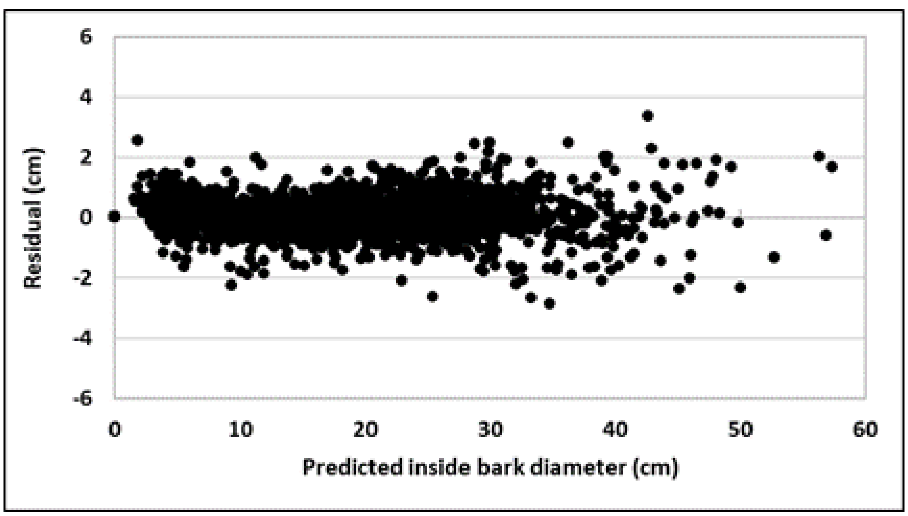
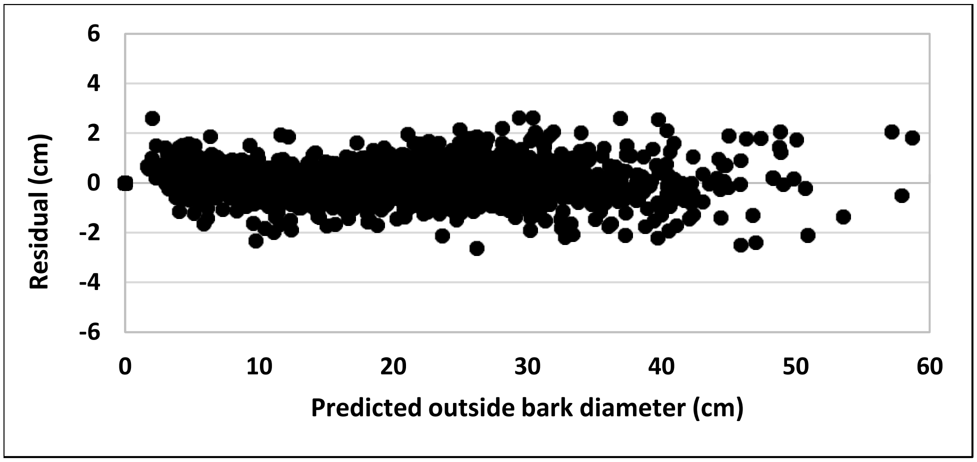


Figure 2. Residuals (observed – predicted) of white spruce plantation inside bark diameters estimated using equation (4) with (top) stand density term (QMD/D) and (bottom) crown ratio plotted against predicted inside bark diameters.

(a)



(b)

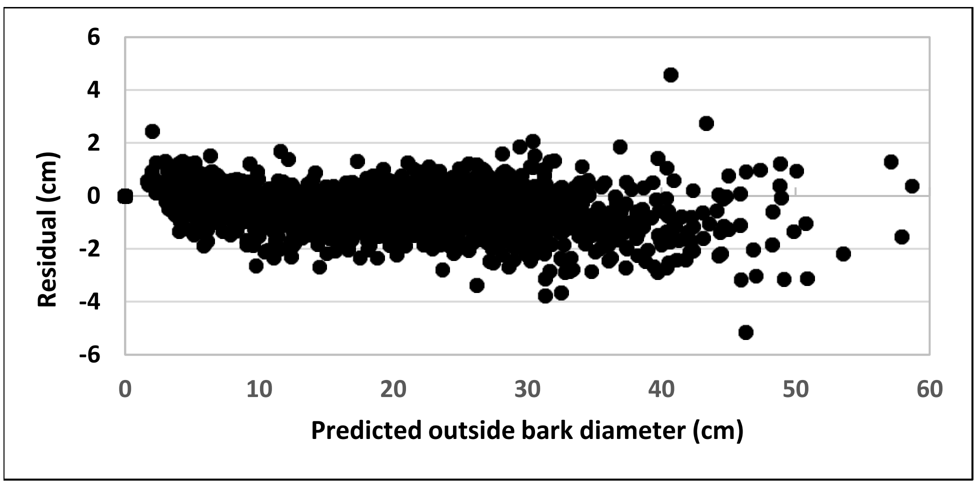
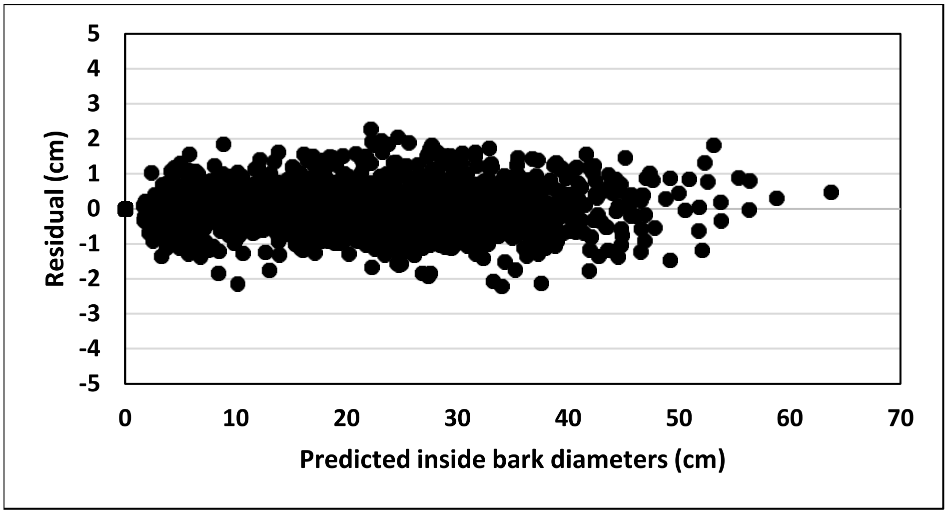


Figure 3. Residuals (observed – predicted) of white spruce plantation outside bark diameters estimated using equation (4) with (a) stand density term (QMD/D) and (b) crown ratio plotted against predicted outside bark diameters.

(a)



(b)

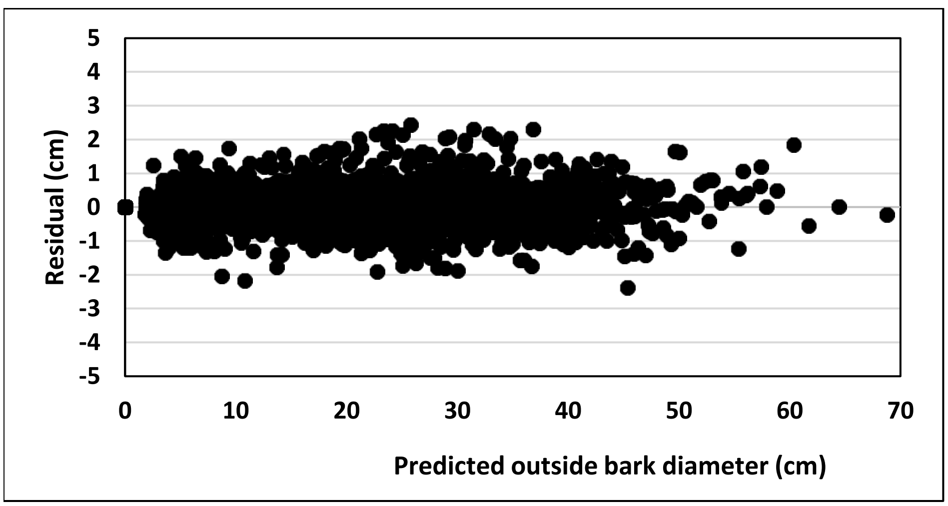


Figure 4. Residuals (observed – predicted) of white pine plantation inside (a) and outside bark (b) diameters estimated using equation (4) with stand density term (QMD/D) plotted against predicted inside and outside bark diameters, respectively.

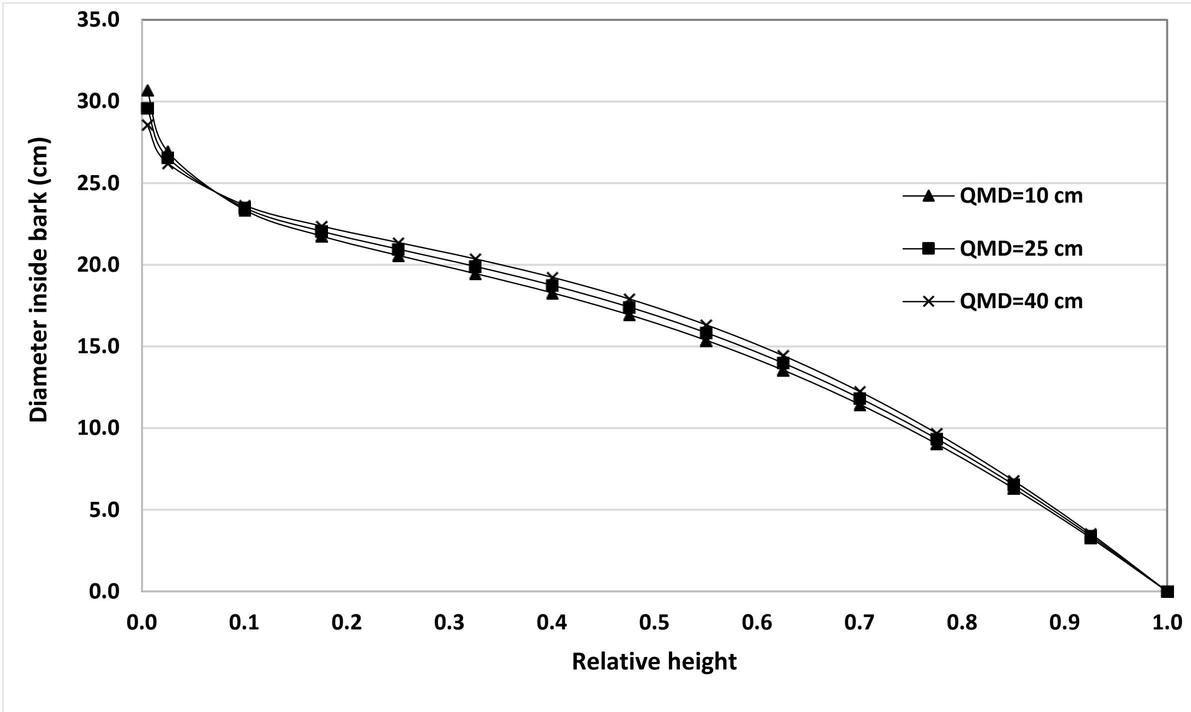
Table 6. Mean biases (cm) (observed – predicted) and their standard deviations in predicting inside and outside bark diameters using equation (4) for stand density and crown ratio of white spruce trees grown in plantations across Ontario.

Relative height	n	With stand density		With crown ratio	
		Bias	Std dev	Bias	Std dev
Inside bark					
0.0≤h/H≤0.1	793	-0.1399	0.5819	-0.1402	0.5827
0.1<h/H≤0.2	202	0.4805	0.5291	0.4794	0.5262
0.2<h/H≤0.3	199	0.4197	0.5294	0.4189	0.5285
0.3<h/H≤0.4	199	0.1055	0.5038	0.1050	0.5051
0.4<h/H≤0.5	199	-0.0826	0.5492	-0.0830	0.5499
0.5<h/H≤0.6	199	-0.2269	0.4261	-0.2272	0.4274
0.6<h/H≤0.7	200	-0.2282	0.4308	-0.2284	0.4307
0.7<h/H≤0.8	202	-0.0572	0.4882	-0.0575	0.4887
0.8<h/H≤0.9	199	0.0969	0.5990	0.0965	0.5994
0.9<h/H≤1.0	394	0.1216	0.4220	0.1214	0.4221
Outside bark					
0.0≤h/H≤0.1	793	-0.1443	0.5888	-0.1447	0.5897
0.1<h/H≤0.2	202	0.4974	0.5637	0.4963	0.5612
0.2<h/H≤0.3	199	0.4388	0.5783	0.4378	0.5776
0.3<h/H≤0.4	199	0.1193	0.5314	0.1187	0.5332
0.4<h/H≤0.5	199	-0.0681	0.5775	-0.0685	0.5786
0.5<h/H≤0.6	199	-0.2451	0.4314	-0.2453	0.4330
0.6<h/H≤0.7	200	-0.2432	0.4375	-0.2434	0.4373
0.7<h/H≤0.8	202	-0.0764	0.5086	-0.0767	0.5091
0.8<h/H≤0.9	199	0.1001	0.6136	0.0997	0.6139
0.9<h/H≤1.0	394	0.1436	0.4349	0.1434	0.4350

Table 7. Mean biases (cm) (observed – predicted) and their standard deviations in predicting inside and outside bark diameters using equation (5) with and without stand density term in the model for white pine trees grown in plantations across Ontario.

Relative height	n	With stand density		Without stand density	
		Bias	Std dev	Bias	Std dev
Inside bark					
0.0≤h/H≤0.1	793	-0.1164	0.5222	-0.1138	0.5205
0.1<h/H≤0.2	205	0.4642	0.5322	0.4652	0.5330
0.2<h/H≤0.3	200	0.2881	0.5875	0.2845	0.5866
0.3<h/H≤0.4	202	-0.0529	0.4395	-0.0559	0.4391
0.4<h/H≤0.5	199	-0.2653	0.4660	-0.2679	0.4651
0.5<h/H≤0.6	202	-0.1862	0.4595	-0.1875	0.4611
0.6<h/H≤0.7	202	0.0355	0.5209	0.0355	0.5207
0.7<h/H≤0.8	199	0.1211	0.5222	0.1221	0.5192
0.8<h/H≤0.9	201	0.0971	0.4385	0.0989	0.4405
0.9<h/H≤1.0	397	-0.0580	0.3968	-0.0581	0.3985
Outside bark					
0.0≤h/H≤0.1	793	-0.1046	0.5075	-0.1030	0.5069
0.1<h/H≤0.2	205	0.4320	0.5961	0.4382	0.5981
0.2<h/H≤0.3	200	0.2846	0.7682	0.2809	0.7697
0.3<h/H≤0.4	202	-0.0164	0.5047	-0.0188	0.5051
0.4<h/H≤0.5	199	-0.2684	0.5100	-0.2700	0.5087
0.5<h/H≤0.6	202	-0.1926	0.5027	-0.1930	0.5034
0.6<h/H≤0.7	202	0.0206	0.5705	0.0211	0.5701
0.7<h/H≤0.8	199	0.1042	0.5556	0.1053	0.5531
0.8<h/H≤0.9	201	0.0948	0.4472	0.0961	0.4475
0.9<h/H≤1.0	397	-0.0424	0.4026	-0.0430	0.4037

(a)



(b)

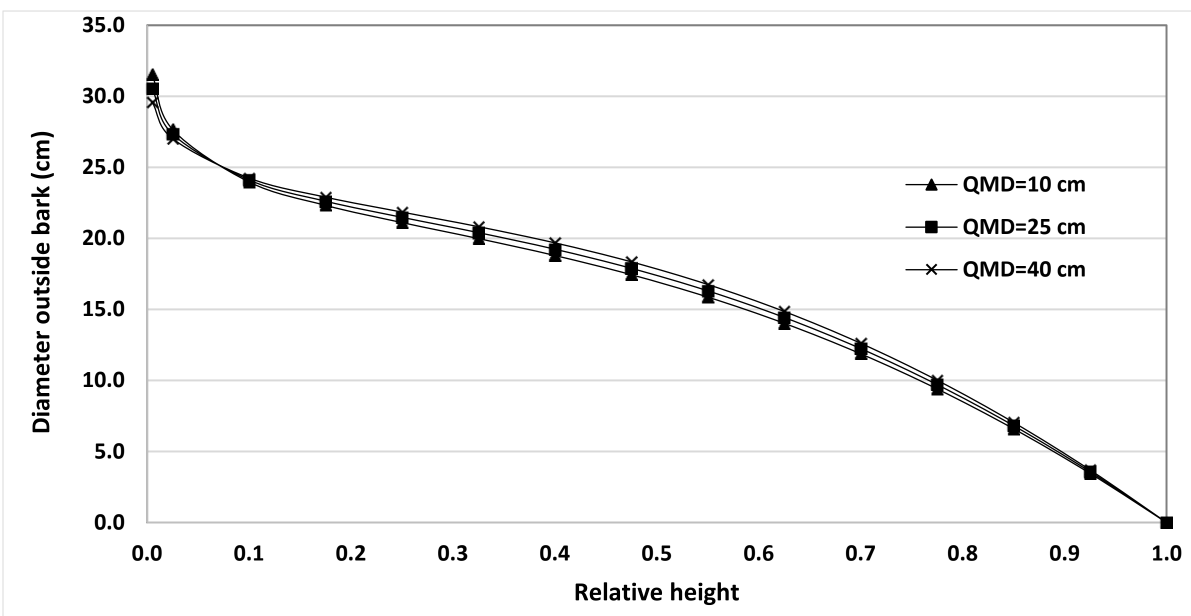
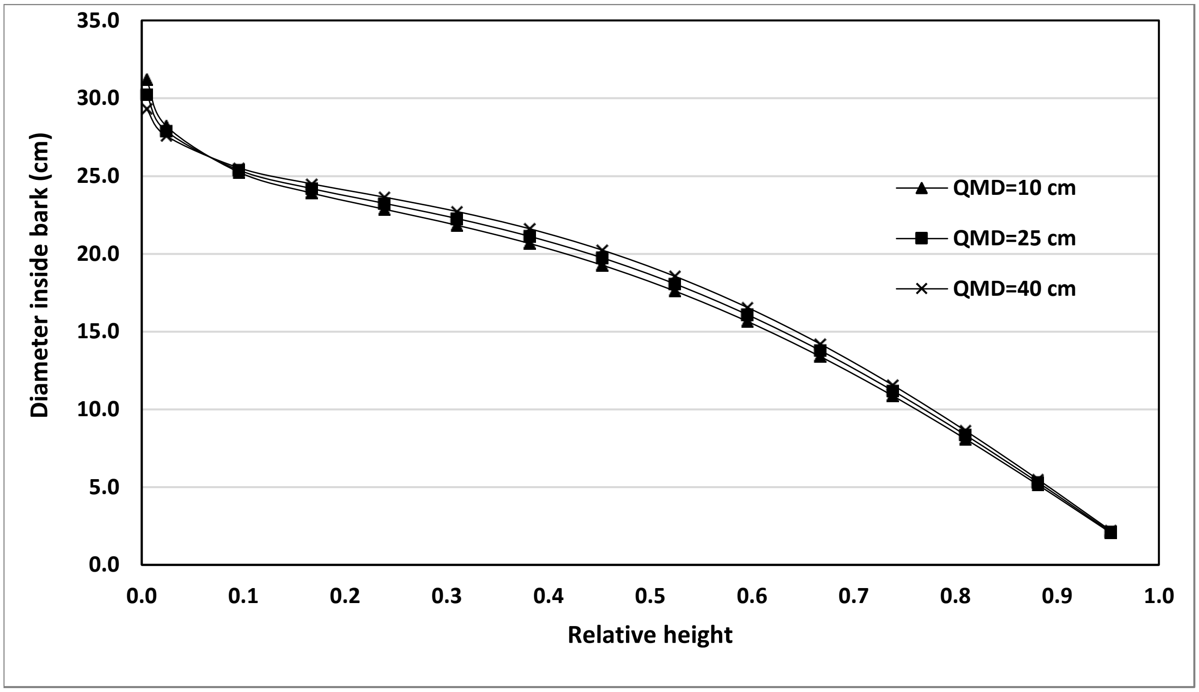


Figure 5. Tree profiles (mean responses) for plantation white spruce generated from equation (4) using DBH=26.0 cm and total height=21.5 m for different QMD values (10, 25, and 40 cm) for (a) inside and (b) outside bark diameters.

(a)



(b)

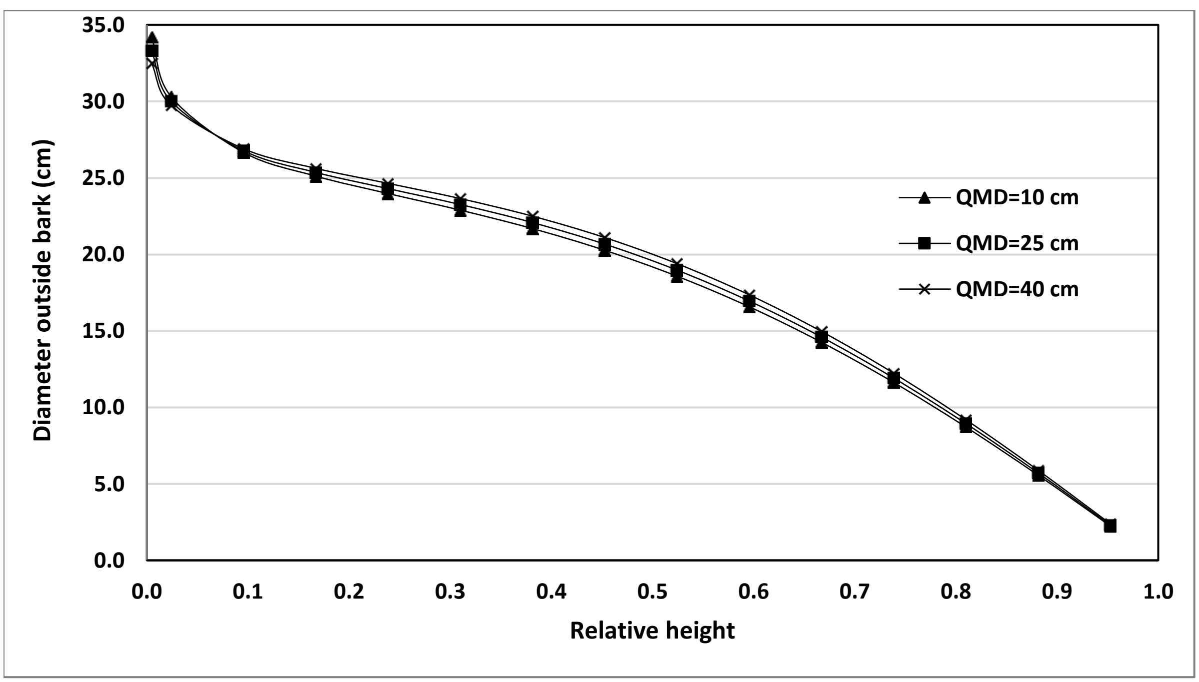


Figure 6. Tree profiles (mean responses) for plantation white pine generated from equation (4) using DBH=28.0 cm and total height=21.0 m for different QMD values (10, 25, and 40 cm) for (a) inside and (b) outside bark diameters.

Discussion

Inside and outside bark taper equations were developed for white spruce and white pine (equations (4) and (5), respectively). QMD/D was selected as the stand density term for both species because the AIC values for those models were lowest. Random effects were added to fixed effects parameters in the models and four of them were significant for white pine, while three were significant for white spruce. Crown ratio was not a significant term for white pine taper models but was for white spruce. However, crown ratio was not significant for white spruce when fitted with three random effects and QMD/D.

Crown density was nonsignificant when fitted with stand density, implying that either stand density or crown ratio (not both) be included in the taper models. Stand density is easier to measure and is readily obtained during forest inventory without additional cost. Crown ratio, however, is difficult to measure and involves extra resources. Stand density was therefore included in the model for both inside and outside bark diameters for white spruce. Furthermore, since stand density and not crown ratio was significant for white pine, stand density was also used in the taper models for that species.

The omission of crown ratio in taper models has occurred for other species. Differences between error sum of squares in models with and without crown ratio for loblolly pine were not significant, and crown ratio was therefore excluded (Burkhat and Walton 1985). When applied to loblolly pine plantations, crown variables improved taper model performance, but the amount did not warrant added cost of data collection (Leites and Robinson 2004). Crown ratio was a significant term in taper models for lodgepole pine (*Pinus contorta* Dougl.), western red cedar (*Thuja plicata* Donn), Douglas fir (*Pseudotsuga menziesii* (Mirb.) Franco), and trembling aspen (*Populus tremuloides* Michx.), but the increased predictive accuracy and fit statistics were marginal and therefore not considered worth the extra cost of measuring crown variables (Muhairwe et al. 1994).

Crown variables were excluded from the models in this study but stand density variables adequately explained the variation in tree taper. In natural black spruce stands, stem density (trees ha^{-1}) significantly explained variation in tree taper (Sharma and Zhang, 2004). Basal area per hectare was also significant in black spruce and jack pine plantations (Sharma and Parton 2009), as was stand planting density in Chinese fir (*Cunninghamia lanceolate*) plantations in southern China (Duan et al. (2016). Overall, stand density terms improve fit statistics and predictive accuracy of taper models.

Recently, Sharma (2020) derived a variable exponent taper model for red pine plantations based on a dimensionally compatible taper model (Sharma and Oderwald 2001), using several functions of stand density to analyze their effects on taper. He found that BA and stem density explained significant variation in taper and improved predictive accuracy for plantation grown red pine trees. Similarly, Sanquetta et al. (2020) included crown ratio, tree hierarchical position and relative spacing in the same taper function (Sharma and Parton 2009) for black wattle trees grown in southern Brazil and found the inclusion of tree position and relative spacing resulted in higher accuracy in estimating total volume and stem diameter.

Overall, crown ratio is affected by stand density, with higher density resulting in a smaller crown ratio values. In this study, both crown ratio and QMD (a function of BA and trees per ha) were significant in explaining the variation in taper of planted white spruce. However, crown ratio became nonsignificant in the presence of QMD. Individually, both variables resulted in almost the same level of fit statistics and predictive accuracy of the taper model for white spruce. Due to the additional resources needed to obtain crown ratio information and the marginal difference in model fit between models with crown ratio and stand density, QMD was selected as the predictor variable for both inside and outside bark taper models in this study.

The taper models presented here are dimensionally compatible and can be applied to data with any units. Diameter at any point along the bole can also be used in place of DBH. This feature is useful when the diameter of a tree cannot be measured at breast height because of an irregular stem (bump, swelling, branch, etc.). In this case, breast height (h_D) is replaced by the height where the reference diameter was measured.

Conclusions

Inside and outside bark taper models were developed for white spruce and white pine plantations. Stand density parameters and crown ratio were assessed, but due to the marginal differences in model accuracy and fit and the increased cost of collecting crown ratio information, density was selected as the model parameter. The quadratic mean diameter — a function of basal area and trees per hectare — was significant in describing taper for both inside and outside bark diameters for both species and was therefore selected as the stand density parameter.

A nonlinear mixed-effects approach was applied in fitting these models. Assuming random effects for four and three of the five parameters significantly improved model fit (AIC and MSE) for white spruce and white pine, respectively. This model can be applied to other tree species by fitting it to species-specific data.

References

Amidon E.L. 1984. A general taper functional form to predict bole volume for five mixed-conifer species in California. Forest Science 30(1): 166–171.

Arias-Rodil M., F. Castedo-Dorado, A. Camara-Obregon and U. Dieguez-Aranda. (2015). Fitting and calibrating a multilevel mixed-effects stem taper model for maritime pine in NW Spain. PLoS ONE 10(12): e0143521.

Avery, T.E. and H.E. Burkhart. 2001. Forest Measurements (5th Edition). McGraw-Hill Education, New York, NY. 456 p.

Burkhart, H.E. and S.B. Walton. 1985. Incorporating crown ratio into taper equations for loblolly pine trees. Forest Science 31(2): 478–484.

Cao, Q.V., H.E. Burkhart, and T.A. Max. 1980. Evaluation of two methods for cubic volume prediction of loblolly pine to any merchantable limit. Forest Science 26(1): 71–80.

Demaerschalk, J.P. and A. Kozak. 1977. The whole bole system: a conditioned dual equation system for precise prediction of tree profiles. Canadian Journal of Forest Research 7(3): 488–497.

Demidenko, E. 2004. Mixed Models: Theory and Applications. John Wiley & Sons, Toronto, ON. 704 p.

Duan, A.G., S. Zhang, X. Zhang and J. Zhang. 2016. Development of a stem taper equation and modelling the effect of stand density on taper for Chinese fir plantations in southern China. PeerJ 4(4): e1929.

Fang, Z., B.E. Borders and R.L. Bailey. 2000. Compatible volume-taper models for loblolly and slash pine based on system with segmented-stem form factors. Forest Science 46(1): 1–12.

Flewelling, J.W. and L.M. Raynes. 1993. Variable-shape stem-profile predictions for western hemlock. Part I. Predictions from DBH and total height. Canadian Journal of Forest Research 23(3): 520–536.

Gray, H.R. 1956. The form and taper of forest-tree stems. Imperial Forest Institute, University of Oxford. Oxford, UK. 78.

Hayden, J., J. Kerley, D. Carr, T. Kenedi and J. Hallarn. 1995. Ontario forest growth and yield program field manual for establishing and measuring permanent sample plots. Ontario Ministry of Natural Resources, Ontario Forest Research Institute, Sault Ste. Marie, ON.

Kozak, A. 1988. A variable-exponent taper equation. Canadian Journal of Forest Research 18(11): 1363–1368.

Larson, P.R. 1963. Stem form development of forest trees. Forest Science 5(S2):a0001–42.

Leites, L.P. and A.P. Robinson. 2004. Improving taper equations of loblolly pine with crown dimensions in a mixed-effects modelling framework. Forest Science 50(2): 204–212.

Max, T.A. and H.E. Burkhart. 1976. Segmented polynomial regression applied to taper equations. Forest Science 22(3): 283–289.

Muhairwe, C.K., V.M. LeMay and A. Kozak. 1994. Effects of adding tree, and site variables to Kozak's variable-exponent taper equation. Canadian Journal of Forest Research 24(2): 252–259.

Newnham, R.M. 1992. Variable-form taper functions for four Alberta tree species. Canadian Journal of Forest Research 22(2): 210–223.

Ormerod D.W. 1973. A simple bole model. The Forestry Chronicle 49(3): 136–138.

Pinheiro, J.C. and D.M. Bates. 1995. Model building for nonlinear mixed effects models. University of Wisconsin, Department of Biostatistics, Madison, WI. Tech. Rep. 91. 11 p.

Reed, D.D. and J.C. Byrne. 1985. A simple, variable form volume estimation system. The Forestry Chronicle 61(2):87–90.

Sanquetta, M.N.I., J.P. McTague, H.F. Scolforo, A. Behling, C.R. Sanqueta and L.N. Schmidt. 2020. What factors should be accounted for when developing a generalized taper function for black wattle trees? Canadian Journal of Forest Research 50(11): 1113–1123.

SAS Institute. 2004. SAS Institute Inc. Cary, NC, USA.

Sharma, M. 2020. Incorporating stand density effects in modeling the taper of red pine plantations. Canadian Journal of Forest Research 50(8): 751–759.

Sharma, M. and H.E. Burkhart. 2003. Selecting a level of conditioning for the segmented polynomial taper equation. Forest Science 49(2): 324–330.

Sharma, M. and R.G. Oderwald. 2001. Dimensionally compatible volume and taper equations. Canadian Journal of Forest Research 31(5): 797–803.

Sharma, M. and J. Parton. 2009. Modeling stand density effects on taper for jack pine and black spruce plantations using dimensional analysis. Forest Science 55(3): 268–282.

Sharma, M. and S.Y. Zhang. 2004. Variable-exponent taper equations for jack pine, black spruce, and balsam fir in eastern Canada. Forest Ecology and Management 198(1–3): 39–53.

Sharma, M., R.G. Oderwald and R.L. Amateis. 2002. A consistent system of equations for tree and stand volume. Forest Ecology Management 165(1 –3): 183–191.

Valenti, M.A. and Q.V. Cao. 1986. Use of crown ratio to improve loblolly pine taper equations. Canadian Journal of Forest Research 16(5): 1141–1145.

Vonesh, E. and V.M. Chinchilli. 1997. Linear and Nonlinear Models for the Analysis of Repeated Measurements (1st edition). CRC Press, Boca Raton, FL. 560 p.

(0k P.R. 23 10 15)
ISBN 978-1-4868-7338-8 (pdf)